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# PREDICTING HYDRAULIC CONDUCTIVITY OF NIGERIAN AGRICULTURAL SOILS USING DIMENSIONAL ANALYSIS

**N. V. NGWANGWA, C. N. MADUBUIKE & S. N. ASOEGWU**

Department of Agricultural Engineering, Federal University of Technology, Owerri, Nigeria

## ABSTRACT

A mathematical model for predicting the hydraulic conductivity of Nigerian agricultural soils in Imo and Abia states at 0 to 15cm depth was presented using dimensional analysis. The model was based on the Buckingham's π theorem using the following soil properties; Bulk density, Porosity, Cation Exchange Capacity, Soil pH, Exchangeable sodium percentage, Organic matter content, Particle density, % Clay, % Silt, % Sand, Acceleration due to gravity, fluid density and depth of soil. The model was validated with the data from the three locations (Soil subgroups) not used in building the model and there was no significant difference between the measured and the predicted hydraulic conductivity values at 5% level of significance. A high coefficient of determination of 0.940 between the measured and the predicted values was also observed.

**KEYWORDS:** Agricultural Soil, Dimensional Analysis, Hydraulic Conductivity, Prediction Equation

## INTRODUCTION

Hydraulic conductivity of soil is the rate at which water flow into the pore spaces or the crack zones of the soil. Agricultural soil is the medium for crop growth, anchorage for plants that contain nutrients, water and air on which plants depend (Ibitoye, 2008). It also refers to the top few centimetres of the land surface and soils having sufficient permeability to maintain drainage and prevent salt accumulation that cause damage to crops is desirable (Brady and Weil, 2002). Vukovic and Soro (1992) stated that restricted movement of water into the soil (low hydraulic conductivity) is desired to prevent water losses from excess drainage. Soil physical, chemical and biological properties affect many processes in the soil including hydraulic conductivity. The hydraulic conductivity of a saturated soil depends mainly on as

The size and distribution of the pores (Boadu, 2000) well as other properties such as soil pH, the density of the soil-water and others. It is imperative to accurately determine these properties for a reliable assessment of the hydraulic conductivity. The measurement of these properties for the determination of hydraulic conductivity of a particular soil could be tedious, time consuming and expensive. Therefore the present study is undertaken to establish a mathematical model for predicting the hydraulic conductivity of Nigerian agricultural soils in Imo and Abia states based on some selected biological, chemical and physical properties of the soil using the dimensional analysis.

## MATERIALS AND METHOD

### Locations and Soil Subgroups of the Study Area

The Nigerian agricultural soils used in this study are from Imo and Abia States bounded by the west by River State, on the east by Ebonyi State, on the north by Anambra States and on the south by Cross River State (Appendix I). Imo State is located between latitudes $4^{o}$ 45'N and $7^{o}$ 15'N and longitudes $6^{o}$ 50'E and $7^{o}$ 25'E with an area of about 5,100 sq. km whereas Abia State is situated between latitudes 4° 40’N and 6° 14’N and longitudes 7° 10’E and 8°E with an area of about 5,243 sq. km. Three major soil groups are found in this study area. These are ferralitic soils covering about 61% of the area; the hydromorphic soils which cover about 31%; and the alluvial soils covering 8% (Madubuike, 2006). Eleven soil sub-groups were identified within these three major groups in Imo and Abia States (Madubuike, 2006). The hydraulic conductivities for the soils were determined based on these 11 soil sub-groups. These 11 soil sub-groups sampled with seven replications each and their locations in the study area are presented in Appendix I.

## Soil Sampling

The eleven (11) locations of the soil subgroups within the study area were sampled randomly from the month of February to March. Core sampler was used to obtain soil samples of about 3kg by weight were taken randomly from seven different pits in each soil subgroup not less than 10meters distance apart from each other by driving a cylindrical iron core sampler into the soil to a depth of 15cm. Samples were properly bagged with polythene bags and taken to the laboratory for the required analyses. All the soils sampled were air dried for six days in the laboratory by spreading them on flat forms except for the parts for the determination of hydraulic conductivity of soils that was air dried for 24hours only (about 1kg of soil from each pit).

## Determination of Model Parameter

Bulk density: Bulk density, $D_b$ was determined insitu using the core method described by Grossman and Reinsch (2002). This was done by driving the core sampler vertically into the soil to 15cm depth in order to fill the sampler. The core sampler was later removed carefully from the hole so as to get an undisturbed soil sample insitu. The weight of the sampler when empty and its weight with the soil sample was recorded as $W_1$ and $W_2$ after trimming the soil extending from both ends of the sampler with a knife,. The volume of the soil sample which is the same as the volume of the core sampler was calculated using Eqn. 1.

$$V = \pi r^2 h \quad (\text{in cm}^3) \tag{1}$$

Where, h and r are the height and radius of the core sampler respectively. Bulk density is calculated as stated in Eqn. 2.

Bulk density,

$$D_b = \frac{W_2 - W_1}{v} \quad (\text{in g/cm}^3) \tag{2}$$

Particle density**:** Particle density, $D_p$ was determined using pycnometer (specific gravity bottle) as described by Ibitoye (2008). This involves cleaning an empty dry 50ml specific gravity bottle weighing say $W_a$. A known quantity of air dried soil was poured inside it and weighed as $W_s$. Distilled water previously boiled and cooled in order to remove gas was used to fill the bottle containing the soil gently so as to remove air between the particles and weighed as $W_{sw}$. The temperature of the contents was determined after a good stirring. Finally, the content of the bottle was removed. The bottle was then filled with boiled, cooled distilled water at the same temperature with that obtained previously and weighed as $W_w$. The particle density was calculated using Eqn. 4:

Density of water,

$$d_w = \frac{W_w - W_a}{50} \quad (\text{in g/cm}^3) \tag{3}$$

Particle density,

$$D_p = \frac{d_w(W_s - W_a)}{(W_s - W_a) - (W_{sw} - W_w)} \tag{4}$$

Porosity: Porosity, P was determined using Eqn. 5.

$$P = \left[\left(1 - {}^{D_b}/_{D_p}\right) \times 100\right] \% \tag{5}$$

P = porosity, %, $D_p$ = particle density, g/cm$^3$ and $D_b$ = bulk density, g/cm$^3$

Exchangeable Calcium and Magnesium: Exchangeable calcium and magnesium determination was done using EDTA titration method as described by Ibitoye (2008). This procedure was done as follows; ten millilitres of soil-water solution sample was pipetted into a 250ml conical flask as $V_2$, 100ml of deionized water added together with 100ml of 20g of potassium cyanide (KOH). Also, 10 drops of 2g potassium cyanide (KCN) was added to the solution followed by 10 drops of 5g hydroxyl ammine hydrochloride ($OHNH_2HCL$). A pinch of calcine indicator was then added into the solution and then the solution was titrated with 0.01 mole EDTA from wine red to deep blue. The titration was repeated three times and the mean value, T calculated. Eqn. (6) gives the calcium ion content, while Eqn. (7) gives the magnesium ion content. Sum of them gave the total cation in the soil.

Calcium,

$$C_a = T \times \text{mol of EDTA} \times \frac{V_1}{V_2} \times \frac{100}{w} \text{ (in } Mg/100g) \tag{6}$$

$$\text{Magnessium}, M_g = (C_a + M_g) - C_a \text{ (In } Mg/100g) \tag{7}$$

Ca, Mg, $V_1$, $V_2$ and w represents total calcium ion in the soil sample; total magnesium ion in the soil sample; volume of final extract, ml; volume of initial titre, ml and weight of sample, grams respectively.

Particle size: Using 2mm mechanical sieve shaker, particle size distribution of less than 2mm fractions was determined using hydrometer method as described by Ibitoye (2008). Hundred millilitre of calgon solution was added into a 250ml beaker containing 50grams of 2mm sieved air dried soil sample and stirred for about 3minutes. The whole suspension was transferred into a sedimentation cylinder and filled to the maximum mark with distilled water having the hydrometer immersed in it. After stirring again vigorously, the hydrometer was lowered carefully into the suspension and hydrometer reading taken after 40 seconds as $R_{40s}$ and the temperature, $R_e$ of the suspension read using a thermometer. The $R_{40s}$ reading was taken simultaneously by two attendants and the readings were taken to ensure that a reliable reading has been obtained. After 2hrs, the hydrometer reading was taken again as $R_{2hrs}$ as well as the temperature, $R_d$ and all were recorded appropriately. After 40seconds, all the sand would have settled and silt and clay remaining in suspension. The procedure was repeated for the blank solution without soil sample and readings taken as $R_a$ for 40seconds and $R_b$ for 2hours. The hydrometer stem reads directly in grams of soil/litre of suspension. To correct the hydrometer reading for temperature, add 0.36g/l for every 1°C above 20 °C and subtract 0.36g/l for every 1°C below 20°C (Ibitoye, 2008). The whole readings were substituted in Eqns. (8-10) so as to get the percentage clay, silt and sand respectively.

$$\% \text{ (Clay } + \text{Silt)} = \left[\frac{(R_{40s} - R_a) + R_e}{wt}\right] \times 100 \tag{8}$$

$$\% \text{ Clay } = \left[\frac{(R_{2hrs} - R_b) + R_d}{wt}\right] \times 100 \tag{9}$$

$$\% \text{ Sand } = 100 - \%(\text{Clay} + \text{Silt}) \tag{10}$$

$R_{40s}$ and $R_{2hrs}$ = hydrometer reading after 40 seconds and 2hours, grams; $R_a$ and $R_b$ = 40seconds and 2hours blank hydrometer reading, grams; $R_e$ and $R_d$ = Temperature readings of the suspension for 40seconds and 2hours, °C; wt = weight of soil sample, grams.

Organic matter content: Organic carbon was determined using Walkley-Black wet oxidation method procedure as described by Nelson and Sommers (1996). Half grams of soil was crushed to a fine material and put in duplicate into 250ml conical flask. Ten millilitres of 0.167 mole of $K_2C_{r2}O_7$ and 20ml of conc. $H_2SO_4$ was added rapidly and immediately the flask was gently swirled until the soil and reagents are mixed. The flask was rotated again gently and allowed to stand on a sheet of asbestos for about 30minutes after adding 100ml of distilled water. Four drops of ferroin indicator was added, titrated with 0.5mole of $fe^{2+}$ and immediately the solution takes a greenish cast and then changes to dark green. Then, the ferrous sulphate was added drop by drop until colour changes sharply from green to brownish red. The procedure was repeated for the blank titration without soil sample. The $K_2C_{r2}O_7$ oxidizes the carbon and the excess $K_2C_{r2}O_7$ is titrated with the Iron (11) solution. In wet oxidation method, all the organic carbon is oxidized. Some resistant group such as ring-compounds (compounds in which their ring itself contained just carbon atoms) are only slightly attacked. It is assumed that an average of 75% of the total organic carbon is attacked and the amount calculated from the titration is multiplied by 100/75 (1.33) to give % organic carbon corrected. It is also assumed that soil organic matters contain 58% of carbon and can be expressed as % organic carbon multiplied by 100/58 (1.724). The percentage organic carbon was calculated using Eqn. 11, where one millilitre solution of 0.167 mole of $K_2C_{r2}O_7$ contains 0.003grams of carbon.

$$\% \text{ Organic Carbon} = (B - T) \times M \times 0.003 \times 1.33 \times 100/wt \tag{11}$$

Where; wt, B, T and M represents weight of soil sample, 0.5g; Blank titre value, 21ml; Sample titre value, 14.67ml and Molarity of $(NH_4)2fe\ (SO_4)6H_20$, 0.48M respectively.

Soil pH: Using a mechanical sieve shaker, 20grams of 2mm sieved air dried soil sample was put into the 100ml beaker, 20ml of distilled water was added making the soil-water ratio to be 1:1 and the whole solution is stirred for about 30minutes vigorously. The solution was left to stand and settle for about 6hrs and the glass electrode of the pH meter immersed deep enough in the clear solution on top of the settled suspension so as to read and record the result shown on the pH meter.

Hydraulic conductivity: The hydraulic conductivity was determined using the falling head permeameter of type ELE 25-0605 cell with manometer tubes which is the laboratory method. One kilogram of air dried (24hours drying) soil samples from each pit were taken to the laboratory mixed with about 500g of water by weight and left to stand for 24hours and using the falling head permeameter; the individual parameters for determining the hydraulic conductivities were got. Hydraulic conductivities were calculated using the Dirksen (1991) formula (Eqn.12).

$$K = \left(\frac{aL}{At}\right) \ln\left(\frac{H_1}{H_2}\right) \quad \text{(in cm/sec)} \tag{12}$$

A, L, a, $H_1$, $H_2$ and t represents cross-sectional area of the cylindrical soil column, $cm^2$; length of soil column, cm; cross-sectional area of the burette through which the percolating fluid is introduced into the system, $cm^2$; initial head of water in the burette, cm; final height of water, cm and time taken to get a head loss, seconds respectively.

**Model Development**

Dimensional analysis was utilized in the model after choosing the major properties of importance influencing hydraulic conductivity of agricultural soils (Table 1) and using them to predict the kind of relationship between them and hydraulic conductivity based on the fundamental dimensions of mass (M), length (L) and time (T).

The following influencing factors were considered for the model development:

- Physical factors/properties: grain size distribution, particle density, bulk density, porosity, fluid density, acceleration due to gravity and soil depth.
- Biological factor/property: organic matter content; and
- Chemical factor/properties: cation exchange capacity, exchangeable sodium percentage and soil pH.

**Table 1: Dimensions of the Selected Variables Influencing Soil Hydraulic Conductivity**

| S/N | Variables | Symbol | Unit | Dimension |
|---|---|---|---|---|
| 1 | Hydraulic conductivity | K | cm/sec | $M^0L^1T^{-1}$ |
| 2 | Soil pH | pH | % | $M^0L^0T^0$ |
| 3 | Acceleration due to gravity | g | cm/sec $^2$ | $M^0L^1T^{-2}$ |
| 4 | Organic matter content | OMC | % | $M^0L^0T^0$ |
| 5 | Exchangeable sodium | ESP | % | $M^0L^0T^0$ |
| 6 | Cation exchange capacity | CEC | % | $M^0L^0T^0$ |
| 7 | Porosity | P | % | $M^0L^0T^0$ |
| 8 | Bulk density | $D_b$ | g/cm $^3$ | $M^1L^{-3}T^0$ |
| 9 | Particle density | $D_p$ | g/cm $^3$ | $M^1L^{-3}T^0$ |
| 10 | Fluid density | $D_f$ | g/cm $^3$ | $M^1L^{-3}T^0$ |
| 11 | Percentage sand | %S | % | $M^0L^0T^0$ |
| 12 | Percentage clay | %Cl | % | $M^0L^0T^0$ |
| 13 | Percentage silt | %Si | % | $M^0L^0T^0$ |
| 14 | Soil depth | H | cm | $M^0L^1T^0$ |

Before developing the model, the following assumptions were made:-

- The gravitational force is constant throughout the locations of the soil subgroups.
- Soil depth of 15cm remained constant throughout the sampling process.
- The ambient air temperature remains constant throughout the experimental area.

Based on the aforementioned assumptions, the dimensional analysis was determined using Burkingham pi theorem. The hydraulic conductivity can be expressed as shown in Eqns. 13-15 below:-

$$K = (pH, OMC, ESP, CEC, P, D_f, D_p, D_b, g, H, \%S, \%Cl, \%Si) \tag{13}$$

Or it can be expressed as:-

$f_1$ (K, pH, OMC, ESP, CEC, P, $D_f$, $D_p$, $D_b$, g, H, %S, %Cl, %Si) = 0 (14)

The total number of variables, n = 14 and number of fundamental dimensions, m = 3.

Then, the number of dimensionless $\pi$–terms = n – m = 14 – 3 = 11.

The equation's dimensionless $\pi$–terms as deduced from Buckingham's $\pi$– theorem is:-

$f_1$ ($\pi_1$, $\pi_2$, $\pi_3$, $\pi_4$, $\pi_5$, $\pi_6$, $\pi_7$, $\pi_8$, $\pi_9$, $\pi_{10}$, $\pi_{11}$) = 0 (15)

In the dimensional analysis, the eight dimensionless terms already observed from Table 1 are excluded but has to be added when the other dimensionless terms had been determined (Ndukwu and Asoegwu, 2011). These already observed dimensionless terms are: - pH, OMC, ESP, CEC, P, %S, %Cl, and %Si.

The $\pi$ terms were determined appropriately based on the Buckingham's $\pi$ theorem and substituting the values of $\pi_1$, $\pi_2$, $\pi_3$,… $\pi_{11}$ in Eqn. (15) gives:-

$$\mathrm{f}_1\left[\frac{\mathrm{K}}{\sqrt{\mathrm{gH}}};\ \frac{\mathrm{D_b}}{\mathrm{D_f}};\ \frac{\mathrm{D_p}}{\mathrm{D_f}};\mathrm{pH};\mathrm{OMC};\mathrm{ESP};\mathrm{CEC};\mathrm{P};\%\mathrm{S};\%\mathrm{Cl};\%\mathrm{Si}\right] \tag{16a}$$

Or

$$\frac{\mathrm{K}}{\sqrt{\mathrm{gH}}} = \mathrm{f}\left[\frac{\mathrm{D_b}}{\mathrm{D_f}};\ \frac{\mathrm{D_p}}{\mathrm{D_f}};\mathrm{pH};\mathrm{OMC};\mathrm{ESP};\mathrm{CEC};\mathrm{P};\%\mathrm{S};\%\mathrm{Cl};\%\mathrm{Si}\right] \tag{16b}$$

$$\pi_1 = \mathrm{f}(\pi_2;\ \pi_3;\ \pi_4;\ \pi_5;\ \pi_6;\ \pi_7;\ \pi_8;\ \pi_9;\ \pi_{10};\ \pi_{11}) \tag{16c}$$

Applying the rule of combination of dimensionless $\pi$ terms to reduce it to a manageable level (Shefii et al., 1996; Ndukwu and Asoegwu, 2011) by multiplication and division gives the following:-

Combining $\pi_2\ and\ \pi_3$ by division gives:

$$\pi_{23} = \pi_2^{-1} \times \pi_3 = \frac{D_f}{D_b} \times \frac{D_p}{D_f} = \frac{D_p}{D_b} \tag{17}$$

Dividing $\pi_4\ by\ \pi_5$ gives:

$$\pi_{45} = \pi_4 \times \pi_5^{-1} = \frac{pH}{OMC} \tag{18}$$

Dividing $\pi_6\ by\ \pi_7$ gives:

$$\pi_{67} = \pi_6 \times \pi_7^{-1} = \frac{ESP}{CEC} \tag{19}$$

Multiplying Eqns. 17, 18 and 19 gives:

$$\Longrightarrow \boldsymbol{\pi_2^a} = \pi_{23} \times \pi_{45} \times \pi_{67} = \boldsymbol{\frac{D_p \times pH \times ESP}{D_b \times OMC \times CEC}} \tag{20}$$

Similarly,

Dividing $\pi_8\ by\ \pi_9$ gives:

$$\Longrightarrow \pi_{89} = \pi_8 \times \pi_9^{-1} = \left(\frac{P}{\%S}\right) \tag{21}$$

Dividing $\pi_{11}$ *by* $\pi_{10}$ gives:

$$\Rightarrow \pi_{10} = \pi_{10}^{-1} \times \pi_{11} = \left(\frac{\%Si}{\%Cl}\right) \tag{22}$$

Multiplying Eqns. 21 and 22 gives:

$$\Rightarrow \boldsymbol{\pi_3^a} = \pi_{89} \times \pi_{10} = \left(\frac{\boldsymbol{P\times\%Si}}{\boldsymbol{\%Cl\times\%S}}\right) \tag{23}$$

Reducing Eqn. (16c) using the rule of combination, gives:

$$\Rightarrow \boldsymbol{\pi_1} = f(\pi_2^a; \pi_3^a) \tag{24}$$

Where,

$$\boldsymbol{\pi_1} = \frac{\boldsymbol{K}}{\sqrt{\boldsymbol{gH}}};\ \boldsymbol{\pi_2^a} = \frac{\boldsymbol{D_p \times pH\times ESP}}{\boldsymbol{D_b\times OMC\times CEC}};\ \ \boldsymbol{\pi_3^a} = \left(\frac{\boldsymbol{P\times\%Si}}{\boldsymbol{\%Cl\times\%S}}\right)$$

Substituting in Eqn. (24) gives

$$\Rightarrow \frac{\boldsymbol{K}}{\sqrt{\boldsymbol{gH}}} = \boldsymbol{f}\left\{\left[\frac{\boldsymbol{D_p\times pH\times ESP}}{\boldsymbol{D_b\times OMC\times CEC}}\right]; \left[\frac{\boldsymbol{P\times\%Si}}{\boldsymbol{\%Cl\times\%S}}\right]\right\} \tag{25}$$

$$\boldsymbol{K} = \sqrt{\boldsymbol{gH}}\ \left\{\left[\frac{\boldsymbol{D_p\times pH\times ESP}}{\boldsymbol{D_b\times OMC\times CEC}}\right]; \left[\frac{\boldsymbol{P\times\%Si}}{\boldsymbol{\%Cl\times\%S}}\right]\right\} \tag{26}$$

Equation (26) is the developed model using dimensional analysis.

## Prediction Equation

The equation predicting hydraulic conductivity is established by allowing one term to vary at a time while keeping the other constant and observing the resulting changes in the function (Ndukwu and Asoegwu, 2011). This was established by plotting the values of $\pi_1$ against $\pi_2^a$; keeping $\pi_3^a$ constant at an average value of 0.5151 and also plotting the values of $\pi_1$ against $\pi_3^a$; keeping $\pi_2^a$ constant at an average value of 2.1104 as shown in Figs (1) and (2) respectively.

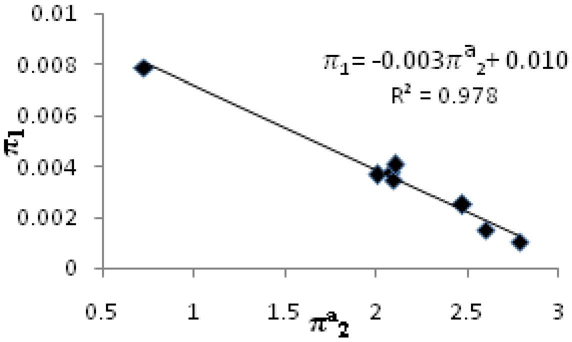

**Figure 1: Plot Of $\pi_1$ Against Dimensionless $\pi_2^a$ With**

**$\pi_2^a$ Constant at average value of 2.1104**

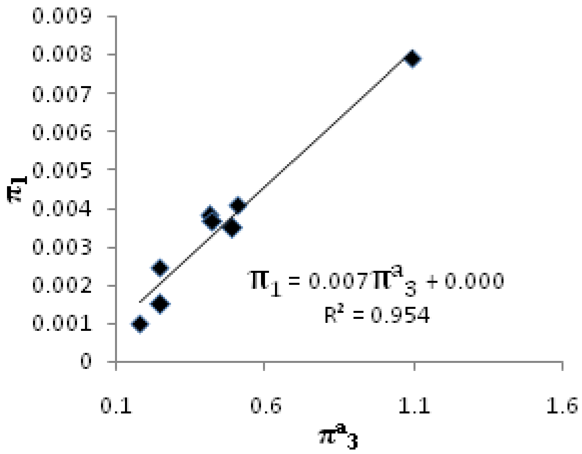

**Figure 2: Plot of $\pi_1$ against Dimensionless $\pi_3^a$ with**

**$\pi_3^a$ Constant at average value of 0.5151**

The linear equations are presented as shown in Eqns (27) and (28) below with coefficient of determination, $R^2 = 0.978$ and 0.954 respectively.

$$\pi_1 = -0.003\pi_2^a + 0.01 \quad (27)$$

$$\pi_1 = 0.007\pi_3^a + 0.000 \quad (28)$$

Thus, from Figs (1) and (2), the plot of the $\pi$ terms forms a plane surface in linear space and according to Ndukwu and Asoegwu (2011); it implies that their combination favours summation or subtraction. Therefore, the component equation is formed by the combination of Eqns (27) and (28).

$$\pi_1 = \frac{K}{\sqrt{gH}} = f_1(\pi_2^a; \pi_3^a) \pm f_2(\pi_2^a; \pi_3^a) + K \quad (29)$$

Note;

At $f_1$; $\pi_3^a$ was kept constant (0.5151) while $\pi_2^a$varies; at $f_2$; $\pi_2^a$ was kept constant (2.1104) while $\pi_3^a$varies.

$$\frac{K}{\sqrt{gH}} = (-0.003\pi_2^a + 0.01) \pm (0.007\pi_3^a + 0.00) \ldots \quad (30)$$

Subtraction combination gives

$$K_1 = (-0.003\pi_2^a - 0.007\pi_3^a + 0.01) \times \left(\sqrt{gH}\right) \quad (31a)$$

Whereas addition combination gives

$$K_2 = (-0.003\pi_2^a + 0.007\pi_3^a + 0.01) \times \left(\sqrt{gH}\right) \quad (31b)$$

Substituting the values of the dimensionless $\pi$ terms from Eqns (20) and (23) into equations (31a and 31b) gives the general equations for hydraulic conductivity prediction using dimensional analysis:

Hence, the predicting equation using dimensional analysis gives:

$$K_1 = \left[-0.003\left(\frac{D_p \times pH \times ESP}{D_b \times OMC \times CEC}\right) - 0.007\left(\frac{P \times \%Si}{\%Cl \times \%S}\right) + 0.01\right]\left[\sqrt{gH}\right] \quad (32a)$$

$$K_2 = \left[-0.003\left(\frac{D_p \times pH \times ESP}{D_b \times OMC \times CEC}\right) + 0.007\left(\frac{P \times \%Si}{\%Cl \times \%S}\right) + 0.01\right]\left[\sqrt{gH}\right] \tag{32b}$$

Where,

$$\pi_2^a = \frac{D_p \times pH \times ESP}{D_b \times OMC \times CEC};\ \pi_3^a = \frac{P \times \%Si}{\%Cl \times \%S}$$

Equations (32a and 32b) give the prediction equation for hydraulic conductivity involving 12 parameters in different combinations using dimensional analysis.

## RESULTS AND DISCUSSIONS

### Model Validation

The mathematical models in Eqns. (32a and 32b) were validated using the values got from the measured selected soil properties from three locations different from the ones used in developing the models so as to know the combination that predicted hydraulic conductivity better. Microsoft Excel 2007 statistical package for Window Vista was used for the statistical analysis based on general linear model (GLM). The predicted and the measured hydraulic conductivity values got from subtraction and addition combinations are presented in Table (2) with subtraction combination having closer values than that predicted by the addition combination. Therefore, the best prediction equation (32a) is given below;

$$K_1 = \left[-0.003\left(\frac{D_p \times pH \times ESP}{D_b \times OMC \times CEC}\right) - 0.007\left(\frac{P \times \%Si}{\%Cl \times \%S}\right) + 0.01\right]\left[\sqrt{gH}\right] \quad . \tag{32a}$$

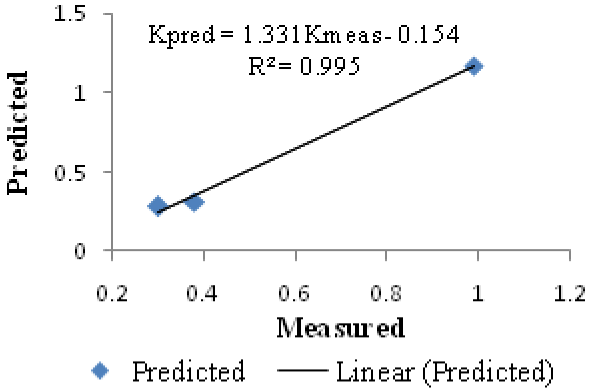

**Figure 3: Plot Showing the Trend of Line for the Three Locations Used in Testing the Model**

**Table 2: Measured and Predicted Hydraulic Conductivity Values from the Three Locations Not Used In Building the Model**

| Location (Soil Subgroup) | Measured (Kmeas) | Predicted ($K_1$pred) | Predicted ($K_2$pred) |
|---|---|---|---|
| Aba (Dystric Ferrasol) | 0.3011 | 0.2788 | 0.6849 |
| Isuochi (Dystric Nitosol) | 0.3796 | 0.3154 | 0.9655 |
| Igbere (Rhodic Ferralsol) | 0.9889 | 1.1671 | 1.2117 |

From Figure 3; it can be observed that the measured value and the predicted value has a very high correlation with $R^2$ value of 0.995 with a standard error of 0.048 between the measured and the predicted value which is less than 1% of the average value of the measured hydraulic conductivity and with standard deviation of 0.50 and 0.37 for predicted and measured hydraulic conductivity values respectively.

Figure 4 shows the trend of line between the measured and the predicted hydraulic conductivity values for all locations in the study area. Also the validity of the model equation was examined by testing if the intercept and the slope were statistically significantly

Different from 0 and 1.0 respectively in the 1:1 model equation (Simonyan et al., 2010).The slope was found not to be significant at 5%. The regression equation obtained by the least square method is:

$$K_{pred} = 1.331K_{meas.} - 0.154 \ (R = 0.995) \quad (34)$$

Where: Kpred and Kmeas represents predicted hydraulic conductivity and measured hydraulic conductivity respectively.

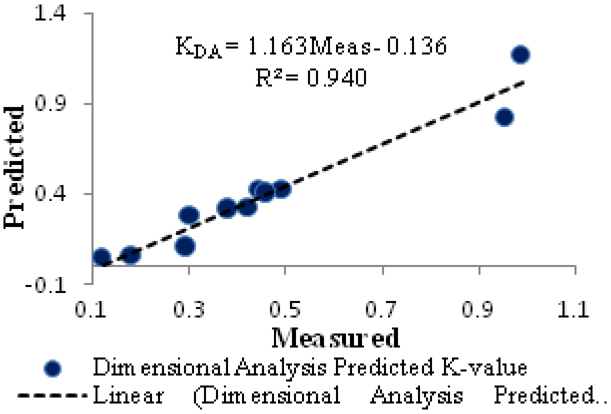

**Figure 4: Plot Showing the Trend of Line for All the Locations**

From Table (2), the model developed using dimensional analysis under-predicted hydraulic conductivity for the Dystric soil-subgroups by between 7.4 – 16.9% and Over-estimated it by 18.0% for Rhodic soil sub-group. This might be as a result of changes in time of sampling and experimentation, microbial activity change during the time of test and the differences in the soil sub-group characteristics in terms of Dystric Ferralsol (Aba) and Dystric Nitosol (Isuochi) possessing classic red soil with high iron, strong acidity, and low organic matter content with moderate resilience (elasticity) according to FAO (2014) classification. It was observed that the difference between actual (measured) and predicted values of hydraulic conductivity were below 20% which implies a good model.

## CONCLUSIONS

A mathematical model was presented using dimensional analysis based on the Buckingham's π theorem. A functional relationship between some soil properties and hydraulic conductivity was established. The model was validated with data from the three locations (Aba, Isuohi and Igbere) not used in building the model. There was no significant difference between the measured and predicted hydraulic conductivity values at 5% level of significance. The results showed a high coefficient of determination ($R^2 = 0.995$) and the difference between the actual (measured) and the predicted hydraulic conductivity values were below 20% which implies good a model.

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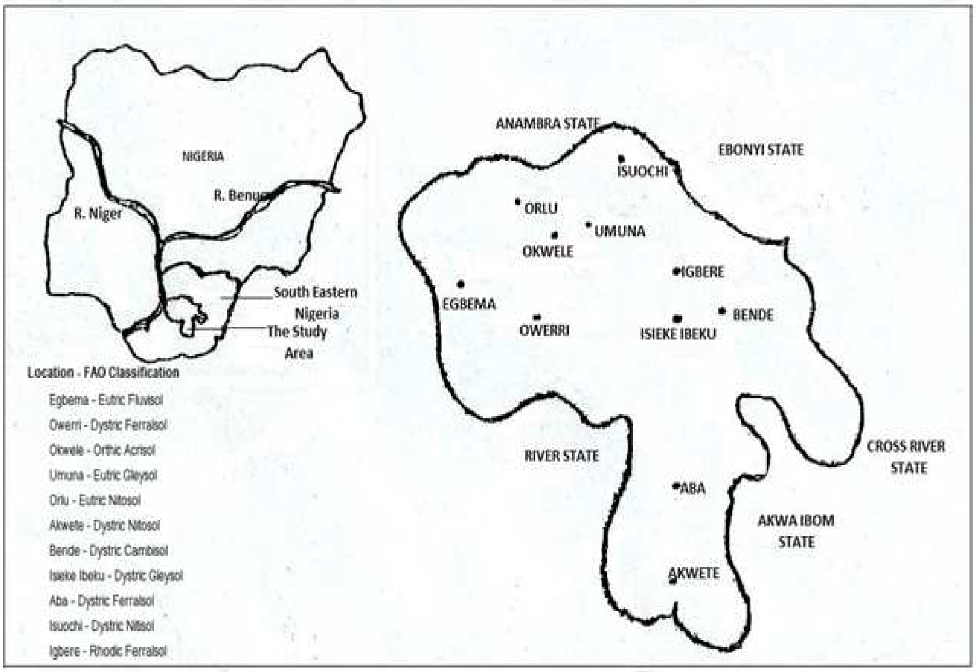
# APPENDICES

## Appendix I



Map Showing the Study Area in Nigeria